Journal of Engineering Physics and Thermophysics, Vol. 74, No. 4, 2001

KINETICS OF THE TEMPERATURE FIELDS IN A LIGHT-SCATTERING LAYER HEATED BY A RADIANT FLUX

A. P. Ivanov and É. I. Vitkin

UDC 536.33

The process of heating and cooling down of a light-scattering and absorbing plane-parallel layer irradiated by a light pulse is discussed. Results are obtained by solving numerically the differential equations of radiation transfer and heat conduction. By the method of finite-difference approximation of space derivatives the problem is reduced to a system of ordinary differential equations with respect to time for the temperature at each point of the layer. Results of calculations for different durations and powers of pulses and different optical and thermophysical parameters of the medium are reported. A comparison is made of the characteristics of temperature processes in the layer on the source side of incidence of a light pulse and on the opposite side.

The majority of objects and media in which it is necessary to investigate thermal conditions are light-scattering. These are, for instance, the materials used in thermal and nuclear power engineering and in the pharmaceutical, chemical, and aerospace industries, as well as the atmosphere and the aqueous and solid mass of the earth, where heat transfer is the governing process in forming the climate of our planet.

A large number of works are devoted to different problems of heat transfer, including the issues of heat transfer in translucent and scattering media. As an example, we can mention several monographs and papers [1–5], the number of which can be brought to many hundreds. However, all of them cause dissatisfaction because of the impossibility of their real application to a wide spectrum of problems owing to complicated mathematical calculations. Only in recent years, in connection with the rapid development of computer engineering, have quantitative calculations in problems of the kinetics of heating of light-scattering materials with allowance for radiant energy transfer become a reality. In particular, in [6–10], highly diverse issues in this field were considered: first of all, the processes of heating of the layer by external heat sources and by grids located inside it with different optical, thermophysical, and geometric characteristics; in one- and multicomponent media; with/without the spectral dependence of optical parameters. The temperature and radiation characteristics and the times of establishment of the steady-state conditions were analyzed. But everywhere consideration was given to the case of heating of a plane layer on two sides, which is accomplished in practice when one is up against the problem of more or less uniform heating of a medium. However, there are many examples of unilateral heating of a light-scattering medium by a radiant flux (solar heating of various objects, laser irradiation in medicine, nuclear explosion, and so on). The indicated case is considered in the present work.

Mathematical Formulation of the Problem. We consider a horizontal, plane-parallel, light-scattering and absorbing layer of thickness x_0. The medium is a "gray" body, i.e., its optical properties are independent of the wavelength of light. Let a radiant flux of irradiance W be incident on the layer normal to its surface. If this radiation were that of a black body, we could assign the following temperature to it:

B. I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 4, pp. 134–137, July–August, 2001. Original article submitted December 12, 2000.

 1062-0125/01/7404-1024$25.00

$$T_W = \left(\frac{W}{\sigma}\right)^{1/4} , \quad (1)$$

where σ = 5.67·10^{-8} W·m^{-2}·K^{-4} is the Stefan–Boltzmann constant. The propagation of radiation in a turbid medium is characterized by the equation of transfer [11, 12]. Since in the present problem it is important to know the integral characteristics of radiation rather than the angular ones for an analysis of the kinetics of heating of the medium, we use the two-flow approximation of the theory of transfer that describes well the irradiance distribution in a plane-parallel layer [11–13]. If the radiation field in the medium is assumed to be diffuse, then, according to [13], for the spectrally integral downward $E_\downarrow$ and upward $E_\uparrow$ fluxes of radiation in the considered layer we can write the following differential equations with allowance for thermal radiation of the medium:

$$\frac{dE_\downarrow}{kdx} = 2\,(\sigma T^4 - E_\downarrow) + \frac{2\Lambda\varphi}{1-\Lambda}\,(E_\uparrow - E_\downarrow)\,, \quad \frac{dE_\uparrow}{kdx} = -\,2\,(\sigma T^4 - E_\uparrow) - \frac{2\Lambda\varphi}{1-\Lambda}\,(E_\downarrow - E_\uparrow)\,. \quad (2)$$

Here $\Lambda = s/(k+s)$ and $\varphi = (3 - x_1)/8$.

The heat-conduction equation is as follows:

$$\frac{\partial T}{\partial t} = \frac{k}{c\rho}\,(E_\downarrow + E_\uparrow - 2\sigma T^4) + \frac{\eta}{c\rho}\frac{\partial^2 T}{\partial x^2}\,. \quad (3)$$

The boundary conditions for the fluxes are

$$E_\downarrow\,(t, x=0) = W\,(t)\,, \quad E_\uparrow\,(t, x = x_0) = 0\,, \quad (4)$$

and for the temperature of the medium are

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = \alpha\,(T - T_0)\,; \quad \left.\frac{\partial T}{\partial x}\right|_{x=x_0} = -\,\alpha\,(T - T_0)\,, \quad (5)$$

where $\alpha = h/\eta$ and T_0 is the initial temperature of the layer. Solution of the problem is reduced to a numerical analysis of the system of differential equations of heat conduction (3) and radiation transfer (2) and is carried out on a personal computer under a specially developed program. It is based on the finite-difference approximation of space derivatives that reduces the problem to a system of ordinary differential equations with respect to time for the temperatures at each point of the considered layer [14, 15].

Analysis of the Results Obtained. We will consider the relative change in the layer temperature $\Delta T/T_0 = (T - T_0)/T_0$ in the process of action of external radiation and after its cessation and also certain time characteristics of the process. So as not to clutter up the problem, we perform an analysis at the inlet to the medium ("for reflection") and at the outlet from it ("for transmission") without considering the structure of the fields in the medium. The initial parameters are as follows: $c\rho$ = 10^6 J·m^{-3}·K^{-1} (this is the characteristic quantity for many media); k = 100 m^{-1}; s = 300 m^{-1}; φ = 0.05. Figures 1 and 2 present the relative changes in the temperature as functions of the time (in a logarithmic scale) for different characteristics of the medium and of the incident radiation.

Figure 1 corresponds to the case where the incident-radiation energy A is constant but the duration of rectangular pulses is different. Here, it is natural that the shorter the pulse, the higher its power W. Figures 1 and 2 present data for pulse powers of 10^3, 10^5, 10^7, and 10^9 W·m^{-2}. According to (1), they correspond to black-body temperatures of 364, 1152, 3644, and 11,523 K.

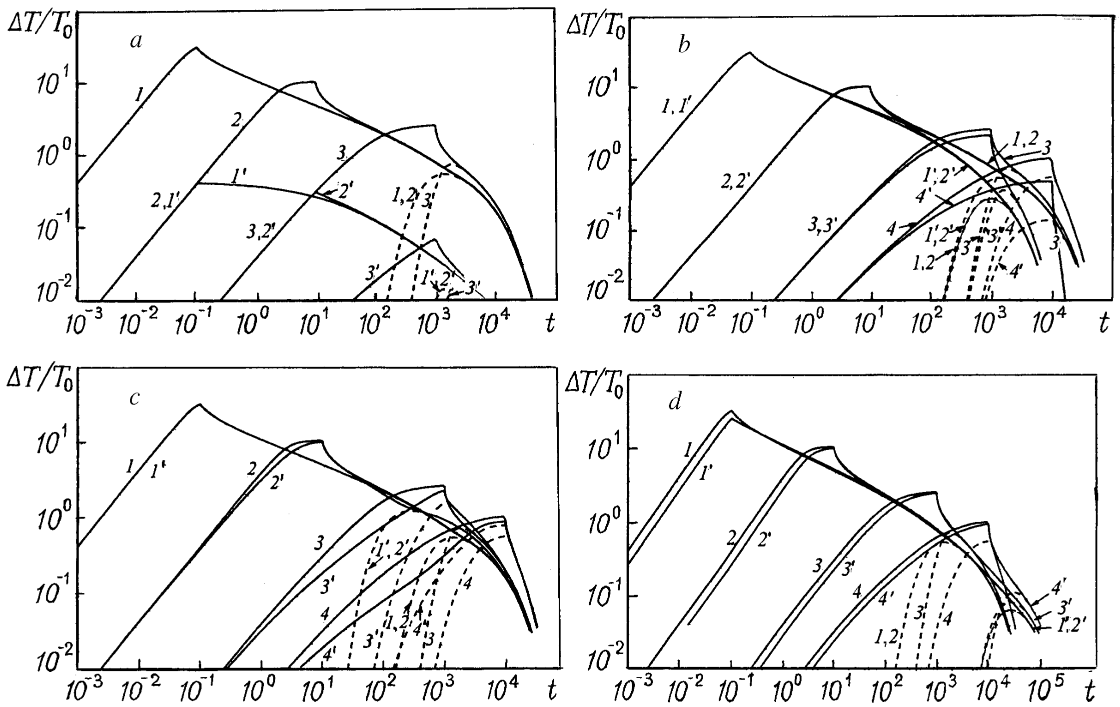

Fig. 1. The plots of $\Delta T/T_0$ vs. t for the constant energy of an incident pulse A but for its different durations t_p [solid lines, "for reflection," dashed lines, "for transmission": 1 and 1′) t_p = 0.1 sec; 2 and 2′), 10; 3 and 3′) 10^3; 4 and 4′) 10^4]: a) η = 0.1 $W \cdot m^{-1} \cdot K^{-1}$; x_0 = 0.03 m; α = 0; unprimed figures, $A = 10^8$ $W \cdot m^{-2} \cdot$sec, primed figures, 10^5; b) $A = 10^8$ $W \cdot m^{-2} \cdot$sec; η = 0.1 $W \cdot m^{-1} \cdot K^{-1}$; x_0 = 0.03 m; unprimed figures, α = 0, primed figures, 100 m^{-1}; c) $A = 10^8$ $W \cdot m^{-2}$sec; x_0 = 0.03 m; α = 0; unprimed figures, η = 0.1 $W \cdot m^{-1} \cdot K^{-1}$, primed figures, 1; d) $A = 10^8$ $W \cdot m^{-2} \cdot$sec; η = 0.01 $W \cdot m^{-1} \cdot K^{-1}$; α = 0; unprimed figures, x_0 = 0.03 m, primed figures, 0.12. t, sec.

Figure 1a provides data for two values of A. It is seen that heating "for reflection" is greater than that "for transmission," and for it to begin a sufficient time for heat transfer is required. Expansion of a pulse (decrease in the power) is always related to a decrease in the maximum temperature in heating. It is characteristic that in the presented scale the heating in the initial stage is always a linear process. The slope of the curves "for transmission" is larger than the slope "for reflection" since the latter occurs in larger times. In the early stage, curve 2 coincides with curve 1′ and curve 3 coincides with 2′ because at these instants the pulse powers are equal. The longer the pulse, the slower the process of heating "for reflection". In the final stage of heating the medium by a longer pulse, the temperature of the medium is higher than that in heating by a short pulse since in the latter case the layer has already cooled down. At large times of cooling down, the temperature at the boundaries of the layer and, consequently, inside it is equalized and all its sections cool down, having the same temperature. Here, the duration of the pulse is insignificant. All is determined just by its energy.

Figure 1b illustrates the influence of the coefficient of heat transfer between the layer and the environment on the kinetics of the thermal process. The regularities demonstrated in Fig. 1a are retained here (as in the subsequent figures), but additional features are also observed. If small times are considered, the conductive mechanism of heat transfer from the heated layer to the surrounding medium is not manifested on the source side of the incidence of external radiation. With increase in t, provided t_p is large, heating slows down

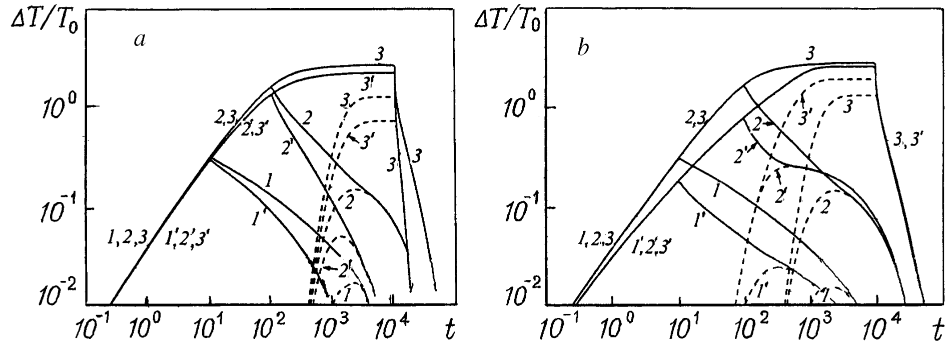

Fig. 2. The plots of $\Delta T/T_0$ vs. t for a constant pulse of an incident power W but for different durations t_p [solid lines, "for reflection," the dashed lines, "for transmission"; 1 and 1′) t_p = 10 sec; 2 and 2′) 100; 3 and 3′) 10^4]: a) $W = 10^5$ W·m^{-2}; η = 0.1 W·m^{-1}·K^{-1}; x_0 = 0.03 m; unprimed figures, α = 0, primed figures, 100 m^{-1}; b) $W = 10^5$ W·m^{-2}; x_0 = 0.03 m; α = 0; unprimed figures, η = 0.1 W·m^{-1}·K^{-1}, primed figures, 1.

with increase in α. The heating "for transmission" begins with a great delay, and here the influence of α is always considerable. For larger α, the heating delays owing to the heat transfer by conduction and the values of $\Delta T/T_0$ will be lower.

We consider the influence of the second thermophysical characteristic, i.e., η (Fig. 1c). At small times, the mechanism of heat conduction does not work "for reflection" and $\Delta T/T_0$ is the same for all dielectrics (η = 0.1 W·m^{-1}·K^{-1}) and for semiconductors (η = 1). If t_p is large, then for η = 1, owing to the more rapid heat transfer into the layer, as compared to that for η = 0.1, the heating "for reflection" is slower and the values of $\Delta T/T_0$ are always smaller. On the contrary, the heating "for transmission" will be more rapid and efficient.

Figure 1d presents the kinetics of the thermal process for two layer thicknesses. The values of k and s are sufficiently great. Therefore, light is distributed in the superficial layer of the medium and the radiation density does not depend on the layer thickness. The material itself is dielectric; therefore, heat transfer into the layer is a slow process. This leads to the fact that the differences in $\Delta T/T_0$ are slight for large and small x_0 in heating of the upper surface. The small decrease in the heating of the surface of a thick layer is related to the greater heat transfer to its depth. After cessation of the action of a pulse, the more heated layer emits intense radiation. As a result, after a small time interval the temperatures of the upper surface of the thin and thick layers are equalized and decrease following the same law. However, at large times the thick layer cools down more slowly than the thin one owing to the great amount of energy accumulated in the heating stage. Heating "for transmission" in the thick layer is, naturally, a slower process than in the thick one. Since the thick layer accumulates a greater amount of thermal energy than the thin one, it cools down more slowly.

Figure 2 corresponds to the case of constant power of the external source $W = 10^5$ W·m^{-2} with its different duration. As previously, the influence of α (Fig. 2a) and η (Fig. 2b) on the kinetics of heating and cooling down of the layer is analyzed. Naturally, at the initial instants, the kinetics of heating "for reflection" will be the same for different t_p as long as $t < t_p$. The longer the pulse, the closer the regime of heating to the steady-state one. In heating "for transmission," the curves $\Delta T/T_0 = f(t)$ rise over one another with increase in t_p. The larger t_p, the earlier the heating begins. If "for reflection" the heating ceases immediately after a pulse, "for transmission" the process of heating and cooling down is described by a smooth curve. Here, the heating "for transmission" can begin considerably later after cessation of the action of an external light pulse (especially for small t_p), which is related to the slow movement of a "thermal wave" into the layer.

With increase in η (Fig. 2b) owing to the faster heat release into the layer, the heating "for reflection" with increase in t_p becomes slower, while the heating "for transmission" becomes quicker. For large t_p,

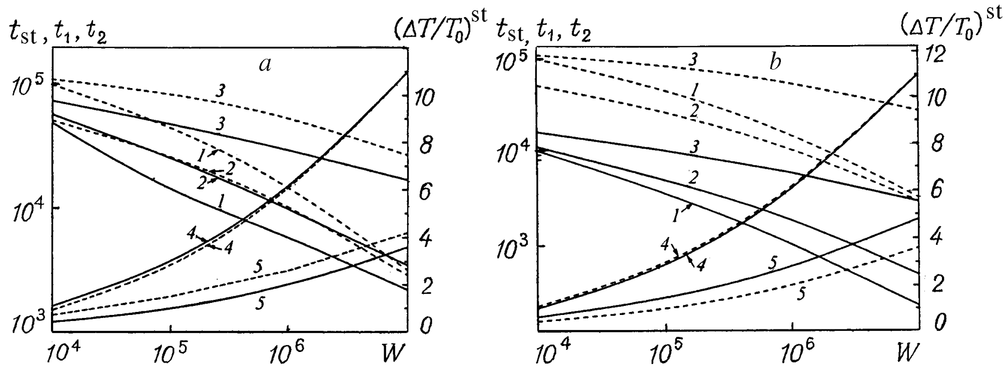

Fig. 3. The plots of t_{st}, t_1, t_2, and $(\Delta T/T_0)^{st}$ vs. W for $\alpha = 0$ [1) t_{st}; 2) t_1; 3) t_2; 4) $(\Delta T/T_0)^{st}_{\uparrow}$; 5) $(\Delta T/T_0)D^{st}_{\downarrow}$: a) $\Lambda = 0.9$; $\alpha = 0$; $x_0 = 0.12$ m, solid lines, $\eta = 0.1$ $W{\cdot}m^{-1}{\cdot}K^{-1}$; dashed lines, 1; b) $\Lambda = 0.9$; $\eta = 0.1$ $W{\cdot}m^{-1}{\cdot}K^{-1}$; solid lines, $x_0 = 0.03$ $W{\cdot}m^{-1}{\cdot}K^{-1}$; dashed lines, 0.12. W, $W{\cdot}m^{-2}$.

when the process of heating approaches the steady-state one and the entire layer is sufficiently heated, the influence of η weakens. With cessation of the action of the external radiation source, the temperature is equalized everywhere and the layer cools down uniformly. The time of equalization of T "for reflection" and "for transmission" depends on t_p.

As calculations show, the influence of x_0 on the kinetics of the thermal field is the same as in Fig. 1d. The heating "for reflection" does not depend on x_0, while in the case of heating "for transmission" the thicker the layer, the slower the heating. But the thick layer cools down longer than the thin layer does. The reasons for this fact have been indicated above.

Let us consider some characteristics of the kinetics of the thermal regime of the layer. We introduce the notions of the time of establishment of the steady-state regime of heating t_{st}, the relative change in the temperature at this time "for reflection" $(\Delta T/T_0)^{st}_{\uparrow}$ and "for transmission" $(\Delta T/T_0)^{st}_{\downarrow}$, and the corresponding times of cooling "for reflection" t_1 and "for transmission" t_2. By t_{st} is meant the time in which the temperature "for transmission" reaches 95% of its limiting value (in this case, T at other points of the layer differs, naturally, from the maximum possible one by less than 5%); by t_1 and t_2 we mean the times in which the values of $(\Delta T/T_0)^{st}_{\uparrow}$ and $(\Delta T/T_0)^{st}_{\downarrow}$ undergo a fivefold decrease.

The influence of the incident-radiation power on t_{st}, t_1, t_2, $(\Delta T/T_0)^{st}_{\uparrow}$, and $(\Delta T/T_0)^{st}_{\downarrow}$ at $T_0 = 300$ K is shown in Fig. 3: for two values of η (a); for two values of x_0 (b). As is seen, as W grows, the times t_{st}, t_1, and t_2 decrease. Here, we always have $t_2 > t_1$ since the temperature on the source side of incidence of external radiation is higher than on the opposite side and in cooling down the colder section is additionally heated by the hot section. With increase in the heat conduction, $(\Delta T/T_0)^{st}_{\uparrow}$ slightly decreases since the process of heat transfer into the layer speeds up. For the same reason $(\Delta T/T_0)^{st}_{\downarrow}$ increases. As the layer thickness changes from 0.03 to 0.12 m, the value of $(\Delta T/T_0)^{st}_{\uparrow}$ remains practically unchanged since in both cases x_0 is sufficiently high. At the same time, $(\Delta T/T_0)^{st}_{\downarrow}$ for a thicker layer will be, naturally, smaller.

NOTATION

x, running coordinate; k, absorption coefficient; s, scattering factor; Λ, probability of survival of a quantum; $\chi(\gamma)$, indicatrix of diffusion (scattering indicatrix); φ, share of the radiation scattered backward by a volume element in its irradiation on one side by diffuse radiation; x_1, first coefficient in expansion of $\chi(\gamma)$ in Legendre polynomials; T, temperature; η, thermal conductivity; c, specific heat; ρ, density; α, coefficient of heat transfer between the layer and its surrounding space; h, heat-release coefficient; t, time; t_p, pulse duration. Subscripts and superscrpts: ↓, downward; ↑, upward; p, pulse; st, steady-state.

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